# A Simple Method for Measuring the Q Value of an NMR Sample Coil 

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A simple method for measuring the $Q$ (quality factor) value of an N MR sample coil based on an impedance matching principle is described. This method has the advantage of utilizing a signal generator and reflection coefficient bridge rather than an expensive high-frequency $Q$ meter and offers an alternative means of measuring the $Q$ value of an N MR sample coil or any other radio frequency coil. © 2000 Academic Press
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The $Q$ value of the sample coil is crucial to the efficiency and sensitivity in the design of an NMR probe circuit (1). The techniques used for measuring the $Q$ value of an NMR sample coil usually require a $Q$ meter or various kinds of bridged-T methods (2-4). The $Q$ meter, especially a higher frequency $Q$ meter, is an expensive instrument, and it is not available in most laboratories. The bridged-T methods also have some limitations when operating at higher frequencies. In addition, it is difficult to measure the $Q$ value of a coil that may be some distance away from the measuring equipment, e.g., an inductor in an NMR probe being used at low temperature and thus mounted in a cryostat (2).
In order to solve this problem a simple method for measuring the $Q$ value of an NMR coil (an inductor) has been developed. An NMR sample coil can be modeled as a pure inductance $L$ in series with an effective resistance $R$. Based on the impedance matching theorem, the impedance matching condition of an NMR sample coil can be obtained with the circuit shown in Fig. 1a (1). For the circuit of Fig. 1a, the impedance between A and C equals

$$
\begin{align*}
Z_{\mathrm{AC}}= & Z_{\mathrm{BC}}+\frac{1}{j \omega C_{1}}=\frac{(R+j \omega L) / j \omega C_{2}}{R+j \omega L+1 / j \omega C_{2}}+\frac{1}{j \omega C_{1}} \\
= & \frac{R}{\left(1-\omega^{2} L C_{2}\right)^{2}+\left(\omega C_{2} R\right)^{2}} \\
& +j \omega \frac{L\left(1-\omega^{2} L C_{2}\right)-C_{2} R^{2}}{\left(1-\omega^{2} L C_{2}\right)^{2}+\left(\omega C_{2} R\right)^{2}}+\frac{1}{j \omega C_{1}} . \tag{1}
\end{align*}
$$

Since the terms $C_{2} R^{2}$ and $\left(\omega C_{2} R\right)^{2}$ are very small and negligible, Eq. [1] may be simplified to

$$
\begin{equation*}
Z_{\mathrm{AC}}=\frac{R}{\left(1-\omega^{2} L C_{2}\right)^{2}}+j \omega \frac{L}{\left(1-\omega^{2} L C_{2}\right)}+\frac{1}{j \omega C_{1}} \tag{2}
\end{equation*}
$$

The equivalent circuit related to Eq. [2] is shown in Fig. 1b, in which

$$
\begin{align*}
& R^{\prime}=\frac{R}{\left(1-\omega^{2} L C_{2}\right)^{2}}  \tag{3}\\
& L^{\prime}=\frac{L}{\left(1-\omega^{2} L C_{2}\right)} . \tag{4}
\end{align*}
$$

In order to satisfy the matching condition $R$, must be set to the characteristic impedance of the connecting coaxial cable, which is $50 \Omega$, by adjusting $C_{2}$ at a fixed frequency:

$$
\begin{equation*}
R^{\prime}=\frac{R}{\left(1-\omega^{2} L C_{2}\right)^{2}}=50 \Omega \tag{5}
\end{equation*}
$$

$C_{2}$ can be obtained from

$$
\begin{equation*}
C_{2}=\frac{1-\sqrt{R / 50 \Omega}}{\omega^{2} L} \tag{6}
\end{equation*}
$$

In order to obtain series resonance in Fig. 1b, $C_{1}$ must be adjusted according to the equation

$$
\begin{equation*}
C_{1}=\frac{1-\omega^{2} L C_{2}}{\omega^{2} L}=\frac{\sqrt{R / 50 \Omega}}{\omega^{2} L} . \tag{7}
\end{equation*}
$$

From Eq. [6] and Eq. [7], we find

$$
\begin{equation*}
\frac{1-\sqrt{R / 50 \Omega}}{C_{2}}=\frac{\sqrt{R / 50 \Omega}}{C_{1}} \tag{8}
\end{equation*}
$$



FIG. 1. (a) The impedance matching circuit of an NMR sample coil. (b) The equivalent circuit of impedance matching of an NMR sample coil.
and it follows that $R$ can be expressed as

$$
\begin{equation*}
R=\frac{50 \Omega}{\left(1+C_{2} / C_{1}\right)^{2}} \tag{9}
\end{equation*}
$$

Using Eq. [6] or Eq. [7], $L$ can be described in terms of $C_{2}$ or $C_{1}$,

$$
\begin{equation*}
L=\frac{1-\sqrt{R / 50 \Omega}}{\omega^{2} C_{2}}, \quad L=\frac{\sqrt{R / 50 \Omega}}{\omega^{2} C_{1}} \tag{10}
\end{equation*}
$$

and the $Q$ value of the coil can now be written as

$$
\begin{equation*}
Q=\frac{\omega L}{R} . \tag{11}
\end{equation*}
$$

By substituting Eq. [9] and Eq. [10] into Eq. [11], the $Q$ value can be represented as

$$
\begin{equation*}
Q=\frac{(1-\sqrt{R / 50 \Omega})\left(1+C_{2} / C_{1}\right)^{2}}{\omega C_{2} 50 \Omega}=\frac{\left(1+C_{2} / C_{1}\right)}{\omega 50 \Omega C_{1}} . \tag{12}
\end{equation*}
$$

At the matching condition, the inductance $L$ and the effective resistance $R$ of an NMR sample coil can be obtained by measuring the $C_{1}$ and $C_{2}$ values of the circuit in Fig. 1a, and hence the $Q$ value of an NMR sample coil can be calculated with Eq. [12].
The instrument assembly for measuring the matching condition of the circuit of Fig. 1a is shown in Fig. 2. The method is based on the comparison of the impedance of the circuit of Fig. 1a with $50 \Omega$ in the Texscan Model RCB-3 reflection coefficient bridge as shown in Fig. 2. A Hewlett Packard Model HP8640B signal generator, which can provide an FM
output by applying a modulation sweep wave voltage from a Hewlett Parkard Model 3311A function generator, provides the input RF source to the bridge in the required frequency range. The output of the function generator also serves as the horizontal sweep for a D10 oscilloscope.
By adjusting $C_{1}$ and $C_{2}$ to the matching condition, the voltage reflection pattern representing the reflection voltage with sweep RF frequency will be displayed on the screen of a D10 oscilloscope. The balance of the RCB-3 bridge is then obtained by turning off the function generator, thus reducing the sweep to zero, and adjusting the fine frequency on the HP8640B to locate the beam on the oscilloscope screen at its maximum position. The frequency displayed on the HP8640B is the resonant frequency of the circuit in Fig. 1a. The values of $C_{1}$ and $C_{2}$ can be calibrated by any traditional method of electronic measurement. Following Eq. [9], or Eq. [10] and Eq. [12], the inductance $L$ and effective $R$ can be calculated, and the $Q$ of the NMR sample coil can be obtained.

Using this arrangement, the result of the measurement for a typical NMR sample coil at 50 MHZ shows $C_{1}=3.2 \rho \mathrm{~F}$ and $C_{2}=30.5 \rho \mathrm{~F}$. By substituting the data into Eq. [9], Eq. [10], and Eq. [12], respectively, $R=0.45 \Omega$ and $L=0.30 \mu \mathrm{H}$. Then the $Q$ value of the NMR sample coil equals 209.4.

Measuring the $Q$ value of this coil directly with a Boonton Type 260-A $Q$ meter at 50 MHZ gives $R=0.47 \Omega, L=0.31$ $\mu \mathrm{H}$, and $Q=212$. This result shows that the difference in $Q$ values by these two methods is only $1.2 \%$. However, the specified accuracy of this model $Q$ meter is $\pm 5 \%$ for directreading $Q$ (5), so that the resulting $Q$ value from the impedance matching method is well within that measurement error.
This method has several special applications. As an example, when an NMR sample coil is immersed in liquid nitrogen for low temperature NMR experiments, it is important to obtain the $Q$ value of the coil at the operating temperature. In


FIG. 2. The assembly of impedance match measurement.


FIG. 3. Measuring the $Q$ value of an NMR sample coil at liquid nitrogen temperature.
this example, the proximity of the Dewar vessel creates severe problems for sufficient space for the coil connectors. Using this new method the sample coil can be connected to a short length (less than $1 / 4 \lambda$ ) of coaxial cable whose capacitance has been measured, and then the coil and part of the coaxial cable can be immersed into liquid nitrogen in a Dewar vessel. By adjusting $C_{1}$ and $C_{2}$ to obtain the matching condition, the $Q$ value of an NMR sample coil can be found. Figure 3 shows the assembly for measuring the $Q$ value of an NMR sample coil immersed in liquid nitrogen in a Dewar vessel. At room temperature the matching condition for a typical coil at 50 MHz was $C_{1}=3.6$ $\rho \mathrm{F}$ and $C_{2}=4.5 \rho \mathrm{~F}$, and the cable exhibits a capacitance of $24 \rho \mathrm{~F}$, which is not affected by the temperature of liquid nitrogen. Therefore, the real $C_{2}$ is equal to $28.5 \rho \mathrm{~F}$, resulting in $R=0.63 \Omega, L=0.32 \mu \mathrm{H}$, and $Q=157.8$. When the Dewar vessel was filled with liquid nitrogen and the matching condition was adjusted $C_{1}=2.3 \rho \mathrm{~F}, C_{2}=5.2 \rho \mathrm{~F}$, and the
real $C_{2}=29.2 \rho \mathrm{~F}$. By calculation $R=0.27 \Omega, L=0.32$ $\mu \mathrm{H}$, and $Q=379.3$. This result shows that the value of $Q$ is significantly different between room temperature and liquid nitrogen temperature.

Since this method is based on impedance matching, the cable connecting the RCB- 3 bridge to the sample coil circuit can be extended to several meters without affecting the matching condition. This distance will be important to avoid interference from the environment to the measuring instruments.

The $Q$ value of an NMR sample coil is an important parameter in the design of an efficient NMR probe circuit. The method reported here offers an additional method for inductor $Q$ measurement, and it can be used to measure the $Q$ of any coil which is used in the radiofrequency range.

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